PROPERTIES OF PAIRING CORRELATIONS IN INFINITE NUCLEAR MATTER

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I. INTRODUCTION

Recently, there has been renewed interest in the pairing problem in neutron matter and neutron-rich nuclei. The superfluid properties of neutron matter are of importance in the study of neutron stars [1], while pairing in neutron-rich systems is of relevance for the study of heavy nuclei close to the drip line [2] and the light halo nuclei [3]. Much effort has gone into calculating the superfluid energy gap in dilute neutron matter [4-9]. Most of these studies have been carried out using pairing matrix elements given by the bare nucleon-nucleon (NN) interaction. Even though it is a long time since Clark et al. [9] showed that density and spin-density fluctuations must be included in the pairing interaction, and there has been much progress in that direction recently [10,11], we will in this contribution focus on selected properties of the pairing problem in infinite neutron and nuclear matter employing only the bare NN interaction.

In this lowest-order approximation to the problem it has been found that all modern NN potentials give nearly identical results for the $^1S_0$ energy gap in dilute neutron matter. One aim of this work is to explain how this can be understood directly from the measured properties of the free nucleon-nucleon (NN) interaction. This is discussed in Section 2.

In Section 3 we discuss, still employing the bare NN interaction, various properties of the pairing wave function. The pairing gap is determined by the attractive part of the NN interaction. In the $^1S_0$ channel the potential is attractive for momenta $k \leq 1.74$ fm^{-1} (or for interparticle distances $r \geq 0.57$ fm). However, the nuclear situation is somewhat different from that of the classical BCS with attractive potentials in the solid state, see e.g., the discussion in Ref. [12]. In the so-called weak coupling regime, where the interaction is weak and attractive, a gas of fermions may undergo a superconducting (or superfluid) instability at low temperatures and a gas of Cooper pairs is formed. This gas of Cooper pairs will be surrounded by unpaired fermions and the typical coherence length is large compared with the interparticle spacing, and the bound pairs overlap. The latter behavior defines also what we will mean with weak-coupling in this work. With weak-coupling we will mean a regime where the coherence length is larger than the interparticle spacing. In the so-called strong coupling limit, the formed bound pairs have only a small overlap, the coherence length is small, and the bound pairs can be treated as a gas of point bosons. One expects then the system to undergo a Bose-Einstein condensation into a single quantum state with total momentum $k = 0$ [12]. For the $^1S_0$ channel in nuclear physics we may actually expect to have two weak-coupling limits, namely when the potential is weak and attractive for large interparticle spacings and when the potential becomes repulsive at $r = 0.57$ fm. In these regimes, the potential has values of typically some few MeV. One may also loosely speak of a strong-coupling limit where the NN potential is large and attractive. This takes place where the NN potential reaches its maximum, with an absolute value of typically ~ 100 MeV, at roughly ~ 1 fm. These properties of the NN potential in the $^1S_0$ channel and their connection with the wave function of the paired state are discussed in Section 3. In that section we will argue, from the properties of the wave function and the calculated coherence length, that fermion pairs in the $^1S_0$ wave in neutron and nuclear matter, will not undergo the above-mentioned Bose-Einstein condensation, since, even though the NN potential is large and attractive for certain Fermi momenta, the coherence length will always be larger than the interparticle spacing.

Concluding remarks and further perspectives for pairing in nuclear systems are given in Section 4.

II. PHASE-SHIFTS AND PAIRING GAP IN INFINITE MATTER

The energy gap in infinite matter is obtained by solving the BCS equation for the gap function $\Delta(k)$.

$$\Delta(k) = -\frac{1}{\pi} \int_0^\infty dk' k'^2 V(k,k') \frac{\Delta(k')}{E(k')}, \tag{1}$$

where $V(k,k')$ is the bare momentum-space NN interaction in the $^1S_0$ channel, and $E(k)$ is the quasiparticle energy given by $E(k) = \sqrt{(\epsilon(k) - \epsilon(k_F))^2 + \Delta(k)^2}$, where $\epsilon(k)$ is the single-particle energy of a neutron with momentum k, and k_F is the Fermi momentum. Medium effects should be included in $\epsilon(k)$, but we will use free single-particle energies $\epsilon(k) = k^2/2m$, where m is the neutron rest mass, to avoid unnecessary complications. The omission of such medium effects is also in line with our omission of screening contributions. Anyway, at the densities considered here, Brueckner-type calculations [7] indicate that in-medium single-particle energies do not differ much from the free ones. The energy gap is defined as $\Delta_F \equiv \Delta(k_F)$. Eq. (1) can be solved by various techniques, some of which are described in Refs. [7,8]. In Fig. 1 we show the results for Δ_F obtained with the CD-Bonn potential (full line) [13], the Nijmegen I and Nimegen II potentials (dotted line and dashed line, respectively) [14]. The results are virtually identical, with the maximum value of the gap varying from 2.98 MeV for the Nijmegen I potential to 3.05 MeV for the Nijmegen II potential. The same insensitivity of the results to the choice of NN interaction was found in Refs. [4,7]. We will now discuss how these results can be understood from the properties of the NN interaction in the $^1S_0$ channel.

A characteristic feature of $^1S_0$ NN scattering is the large, negative scattering length, indicating the presence of of a nearly bound state at zero scattering energy. Near a bound state, where the NN T-matrix has a pole, it can be written in separable form, and this implies that the NN interaction itself to a good approximation is rank-one separable near this pole [15]. Thus at low energies we can write

$$V(k,k') = \lambda v(k) v(k'), \tag{2}$$

where λ is a constant. Then it is easily seen from Eq. (1) that the gap function can be written as $\Delta_F v(k)$, where Δ_F is the energy gap. Inserting this form of $\Delta(k)$ into Eq. (1) one easily obtains

$$1 = -\frac{1}{\pi} \int_0^\infty dk' k'^2 \frac{\lambda v^2(k')}{E(k')}. \tag{3}$$

Numerically the integral on the right-hand side of this equation depends very weakly on the momentum structure of $\Delta(k)$, so in our calculations we could take $\Delta(k) \approx \Delta_F$ in $E(k)$. Then Eq. (3) shows that the energy gap Δ_F is determined by the diagonal elements $\lambda v^2(k)$ of the NN interaction. The crucial point is that in scattering theory it can be shown that the inverse scattering problem, that is, the determination of a two-particle potential from the knowledge of the phase shifts at all energies, is exactly, and uniquely, solvable for rank-one separable potentials [16]. Following the notation of Ref. [15] we have

$$\lambda v^2(k) = -\frac{k^2 + \kappa_B^2}{k^2} \frac{\sin \delta(k)}{k} e^{-\alpha(k)}, \tag{4}$$

for an attractive potential with a bound state at energy $E = -\kappa_B^2$. In our case $\kappa_B = 0$. Here $\delta(k)$ is the $^1S_0$ phase shift as a function of momentum k, while $\alpha(k)$ is given by a principle value integral:

$$\alpha(k) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} dk' \frac{\delta(k')}{k' - k}, \tag{5}$$

where the phase shifts are extended to negative momenta through $\delta(-k) = -\delta(k)$.

From this discussion we see that $\lambda v^2(k)$, and therefore also the energy gap Δ_F, is completely determined by the $^1S_0$ phase shifts. However, there are two obvious limitations on the practical validity of this statement. First of all, the separable approximation can only be expected to be good at low energies, near the pole in the T-matrix. Secondly, we see from Eq. (5) that knowledge of the phase shifts $\delta(k)$ at all energies is required. This is, of course, impossible, and most phase shift analyses stop at a laboratory energy $E_{lab} = 350$ MeV. The $^1S_0$ phase shift changes sign from positive to negative at $E_{lab} \approx 248.5$ MeV However, at low values of k_F, knowledge of $v(k)$ up to this value of k may actually be enough to determine the value of Δ_F, as the integrand in Eq. (3) is strongly peaked around k_F.

The input in our calculation is the $^1S_0$ phase shifts taken from the recent Nijmegen phase shift analysis [17]. We then evaluated $\lambda v^2(k)$ from Eqs. (4) and (5), using methods described in Ref. [18] to evaluate the principle value

integral in Eq. (5). Finally, we evaluated the energy gap Δ_F for various values of k_F by solving Eq. (3), which is an algebraic equation due to the approximation $\Delta(k) \approx \Delta_F$ in the energy denominator.
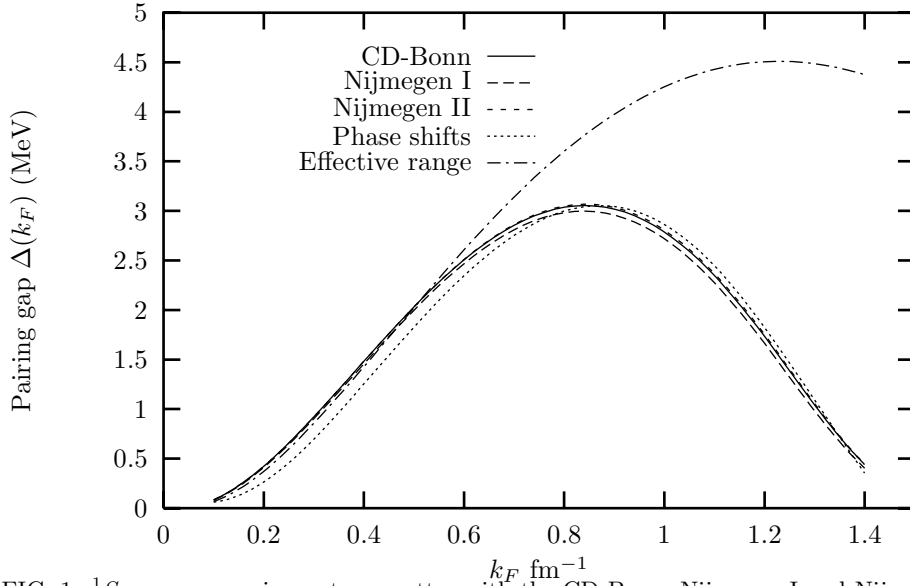

FIG. 1. $^1S_0$ energy gap in neutron matter with the CD-Bonn, Nijmegen I and Nijmegen II potentials. In addition we show the results obtained from phase shifts only, Eqs. (3)-(5), and the effective range approximation of Eq. (6).

The resulting energy gap is plotted in Fig. 1 (full line). As the reader can see, the agreement between the direct calculation from the phase shifts and the CD-Bonn and Nijmegen calculation of Δ_F is, to say the least, satisfying, even at densities as high as $k_F = 1.4$ fm^{-1}. The energy gap is to a remarkable extent determined by the available $^1S_0$ phase shifts. In the same figure we also report the results (dot-dashed line) obtained using the effective range approximation to the phase shifts:

$$ k \cot \delta(k) = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2, \tag{6} $$

where $a_0 = -18.8 \pm 0.3$ fm and $r_0 = 2.75 \pm 0.11$ fm are the singlet neutron-neutron scattering length and effective range, respectively. In this case an analytic expression can be obtained for $\lambda v^2(k)$, as shown in Ref. [16]:

$$ \lambda v^2(k) = -\frac{1}{\sqrt{k^2 + \frac{r_0^2}{4}(k^2 + \alpha^2)^2}} \sqrt{\frac{k^2 + \beta_2^2}{k^2 - \beta_1^2}}, \tag{7} $$

with $\alpha^2 = -2/ar_0$, $\beta_1 \approx -0.0498$ fm^{-1}, and $\beta_2 \approx 0.777$ fm^{-1}. The phase shifts using this approximation are positive at all energies, and this is reflected in Eq. (7) where $\lambda v^2(k)$ is attractive for all k. From Fig. 1 we see that below $k_F = 0.5$ fm^{-1} the energy gap can with reasonable accuracy be calculated with the interaction obtained directly from the effective range approximation. One can therefore say that at densities below $k_F = 0.5$ fm^{-1}, and at the crudest level of sophistication in many-body theory, the superfluid properties of neutron matter are determined by just two parameters, namely the free-space scattering length and effective range. At such densities, more complicated many-body terms are also less important. Also interesting is the fact that the phase shifts predict the position of the first zero of $\Delta(k)$ in momentum space, since we see from Eq. (4) that $\Delta(k) = \Delta_F v(k) = 0$ first for $\delta(k) = 0$, which occurs at $E_{\text{lab}} \approx 248.5$ MeV (pp scattering) corresponding to $k \approx 1.74$ fm^{-1}. This is in good agreement with the results of Khodel et al. [8]. In Ref. [8] it is also shown that this first zero of the gap function determines the Fermi momentum at which $\Delta_F = 0$. Our results therefore indicate that this Fermi momentum is in fact given by the energy at which the $^1S_0$ phase shifts become negative.

Thus, the quantitative features of $^1S_0$ pairing in neutron matter can be obtained directly from the $^1S_0$ phase shifts. This happens because the NN interaction is very nearly rank-one separable in this channel due to the presence of a bound state at zero energy. This explains why all bare NN interactions give nearly identical results for the $^1S_0$ energy gap in lowest-order BCS calculations. However, it should be mentioned that this agreement is not likely to survive in a more refined calculation, for instance if one includes the density and spin-density fluctuations in the effective pairing interaction like in e.g., Refs. [10,11]. Other partial waves will then be involved, and the simple arguments employed

here will, of course, no longer apply. Our reasoning here applies also only to a partial wave where the T-matrix (almost) has a pole, and we have neglected the fact that the phase shifts become negative at higher energies.
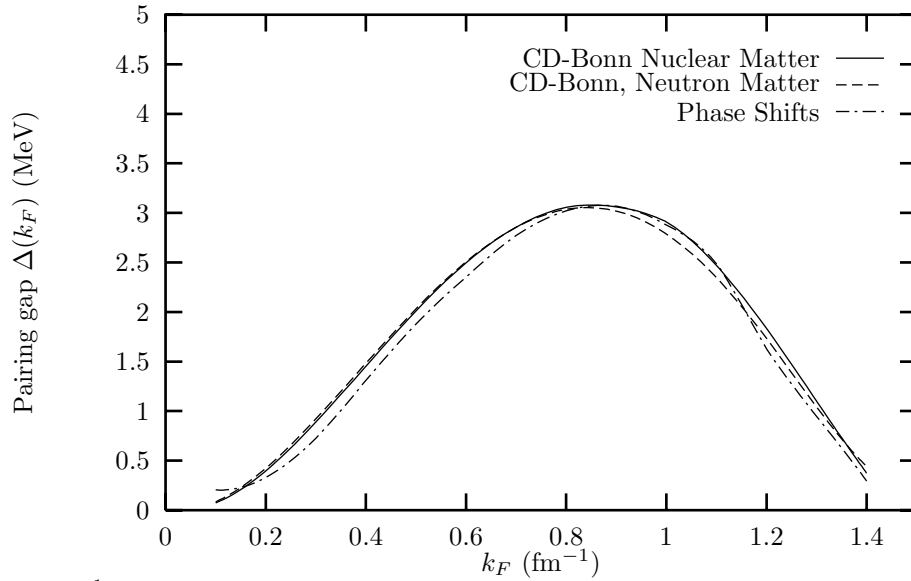

FIG. 2. $^1S_0$ energy gap in nuclear matter calculated with the CD-Bonn potential compared with the direct calculation from the $^1S_0$ np and pp phase shifts. Also shown are the results for neutron matter with the CD-Bonn potential.

The calculation of the $^1S_0$ gap in symmetric nuclear matter is closely related to the one for neutron matter. In fact, with charge-independent forces, like the older Bonn potentials, and free single-particle energies one would, of course, obtain exactly the same results. However, the new potentials on the market are charge-dependent, in order to achieve high quality fits to both np and pp scattering data, and therefore we should in principle solve three coupled gap equations for neutron-neutron (nn), proton-proton (pp), and neutron-proton (np) pairing:

$$\Delta_i(k) = -\frac{1}{\pi} \int_0^\infty dk' k'^2 V_i(k,k') \frac{\Delta_i(k')}{E(k')}, \tag{8}$$

where i=nn, pp and np, and the quasiparticle energy is still given by $E(k) = \sqrt{(\epsilon(k) - \epsilon(k_F))^2 + \Delta(k)^2}$, but the energy gap is now given by

$$\Delta(k)^2 = \Delta_{\rm nn}(k)^2 + \Delta_{\rm pp}(k)^2 + \Delta_{\rm np}(k)^2. \tag{9}$$

Solving these equations, both with the CD-Bonn potential and with the phase shift approximations we get the results shown in Fig. 2. For comparison we have in the same figure plotted the results for pure neutron matter with the CD-Bonn potential (dashed line). From the figure it is clear that the phase shift approximation works well also in this case. As could be expected, the results are very close to those obtained earlier with charge-independent interactions [7].

III. FEATURES OF PAIRING CORRELATIONS IN INFINITE MATTER

An important length scale of e.g., a neutron superfluid is the coherence length. From a microscopic point of view the coherence length represents the squared mean distance of two paired particles (a Cooper pair of neutrons) on top of the Fermi surface. The magnitude of this quantity affects several of the physical properties of a neutron star crust. First of all, neutrons paired in a singlet state form quantized vortices induced by the rotational state of the star. These can pin to the nuclei present in the crust, possibly leading to the observed sudden release of angular momentum known as pulsar glitches. The magnitude of the pinning force depends on the size of the vortex cores, which is equal to the coherence length of the neutron superfluid. A second question is how properties of the neutron superfluid change due to the inhomogeneous environment of a neutron star crust, a problem related to the average thermodynamical properties of neutron matter [1]. The typical dimension of nuclei in the inner crust of a neutron star is $R_N \approx 4-6$ fm. This number is, in an appropriate range of densities, comparable to the coherence length ξ as

estimated from existing BCS calculations. Clearly, the coherence length represents a critical parameter by which one can establish the behavior of an inhomogeneous superfluid. It sets the scale for the possible spatial variation of the pairing properties of the system, and thus plays a role if some inhomogeneities are present in the system at a length scale comparable to it.

The coherence length can be easily evaluated from the wavefunction $\phi(\mathbf{r})$ of the relative motion of the two neutrons in a Cooper pair, $\mathbf{r}$ being the relative coordinate of the two particles. The coherence length ξ is given by

$$\xi^2 = \frac{\int d^3r |\phi(\mathbf{r})|^2 r^2}{\int d^3r |\phi(\mathbf{r})|^2} = \frac{\int_0^\infty dk k^2 |\partial\chi(k)/\partial k|^2}{\int_0^\infty dk k^2 |\chi(k)|^2}, \tag{10}$$

with $\chi(\mathbf{k})$ being the wavefunction of a Cooper pair in momentum space. This equation is particularly suited for numerical computation, since the BCS equations for a uniform system are solved in momentum space, as discussed in the previous section. The wavefunction of the Cooper pair in momentum space is given by (apart from an unimportant normalization constant)

$$\chi(k) = \frac{\Delta(k)}{E(k)}, \tag{11}$$

where $\Delta(k)$ is the k-dependent pairing gap, while $E(k)$ is the energy denominator in Eq. (1).

As mentioned in the introduction, the NN potential in the $^1S_0$ channel yields actually two so-called weak-coupling limits. This happens when the potential is weak and attractive for large interparticle spacings and when the potential becomes repulsive at $r = 0.57$ fm. This corresponds to the Fermi momentum $k_F = 1.74$ fm^{-1} discussed in the previous section in connection with the phase shift analyses. The other weak-coupling limit, i.e., when r is large and the potential tends to zero, corresponds to small values of the Fermi momentum. The pairing gap decays exponentially to zero in the low-density limit [8]. What can be thought of as a strong coupling limit takes place where the NN potential reaches its maximum, at roughly ~ 1 fm, see e.g., Ref. [15] for a discussion of various features of the NN potential. The pairing gaps in Figs. 1 and 2 have their maxima at the density which corresponds roughly to the maximum of the NN interaction in the $^1S_0$ channel.

In the weak-coupling limits, we have that the bound pairs overlap in r-space, or stated differently, that the wave function in Eq. (11) is strongly peaked in momentum space at the value of the corresponding Fermi surface. The coherence length ξ is in this case much larger than the typical interparticle spacing, see e.g., the discussion in Ref. [12]. As stated in the introduction, we will with weak-coupling mean that the gas of Cooper pairs will be surrounded by unpaired fermions and the typical coherence length is large compared with the interparticle spacing, and the bound pairs overlap.

If we have a strong coupling limit, the Cooper pairs at the Fermi surface have only small overlaps, a fact which means in turn that the pair wave function in Eq. (11) extends further out in k-space, or that the Cooper pair is more localized in r-space. The coherence length should then be small, of the order or smaller than the interparticle spacing.

The wave function of Eq. (11) for the various possible coupling regimes is shown in Fig. 3. There we plot $\chi(k)$ for five values of k_F, 0.03, 04, 0.8, 1.2 and 1.4 fm^{-1}, employing the pairing gap from the previous section obtained with the CD-Bonn potential. The phase-shift approximation or the Nijmegen potentials yield essentially the same results.

Clearly, at low values, $k_F = 0.03$ fm^{-1} in Fig. 3, of the Fermi momentum, corresponding to one of the weak-coupling regimes, the wave function is strongly peaked in momentum space. Similarly, for densities where the NN interaction changes from being attractive to repulsive, we have the other weak-coupling regime. The qualitative form of the wave function at $k_F = 1.4$ fm^{-1} resembles much that at low densities. For densities corresponding to r-values where the potential is close to its maximum, $k_F \sim 0.7 - 1.2$ fm^{-1}, one sees that the wave function in k-space is much more spread out, possibly implying that the coherence length is smaller and that the Cooper pairs have only small overlaps.

The fact that the NN interaction in the $^1S_0$ channel is large and attractive at certain values of k_F (up to five times larger than the Fermi energy) and that the wave functions in Fig. 3 extend over several values of k, may lead one to conclude that one could speak of bound fermion pairs which can be treated as a gas of point bosons. One expects then the system to undergo a Bose-Einstein condensation into a single quantum state with total momentum $k = 0$, as discussed in depth in Ref. [12]. However, such a conclusion for singlet pairing in neutron or nuclear matter is wrong. If one calculates the coherence length using Eq. (10) for the above Fermi momenta, one finds that for all Fermi momenta the coherence length is much larger than the typical interparticle spacing. Eq. (10) gives $\xi = 388.0, 4.8, 5.2, 13.2$ and 53.5 fm for $k_F = 0.03, 04, 0.8, 1.2$ and 1.4 fm^{-1}, respectively. Even the smallest values are of the size of the radius of nuclei found in the crust of a neutron star. If one also observes that screening effects yield even larger coherence lengths, see e.g., [19], one can conclude that for singlet pairing in neutron or nuclear matter, the gas of Cooper pairs has a typical coherence length which is large compared with the interparticle spacing, the Cooper pairs overlap and fermion exchange may become dominant.

From Fig. 3 one also notices that for the chosen Fermi momenta, the pair wave function does not vanish at $k = 0$. This behavior is easy to understand if we again employ a rank-one separable interaction. Eq. (11) reads then

$$\chi(k) = \frac{v(k)\Delta(k_F)}{\sqrt{(\epsilon(k) - \epsilon(k_F))^2 + v(k)^2\Delta(k_F)^2}}, \tag{12}$$

which at $k = 0$ simplifies to

$$\chi(0) = \frac{v(0)\Delta(k_F)}{\sqrt{\epsilon(k_F)^2 + v(0)^2\Delta(k_F)^2}}, \tag{13}$$

where λ of Eq. (2) is set equal to one. The $v(0)$ part of the potential can in turn be determined directly from the scattering matrix at $k = 0$. In that limit the scattering matrix equals $-a_0$, where a_0 is the scattering length, $a_0 = -18.8 \pm 0.3$ fm for the neutron-neutron potential. For a rank-one separable potential, the on-shell scattering matrix at $k = 0$ is given by [15]

$$T(k = 0) = -a_0 = \frac{v(0)^2}{1 + \frac{2}{\pi}\int_0^\infty dq v(q)^2}. \tag{14}$$

If the $^1S_0$ channel really has a bound state at $k = 0$, the denominator should diverge, which in turn means that the scattering length should be $a_0 = -\infty$. The fact that the scattering length is finite implies that $v(0)$ is finite. Eq. (13) can be rewritten as

$$\frac{\chi(0)^2\epsilon(k_F)^2}{\Delta(k_F)^2(1 - \chi(0)^2)} = v(0)^2. \tag{15}$$

When $k_F \to 0$, the gap behaves asymptotically as [8]

$$\Delta(k_F) \sim 8\epsilon(k_F)e^{-1/\gamma-2}, \tag{16}$$

where $\gamma = -2k_f a_0/\pi$. Inserting Eq. (16) into Eq. (15) then yields that the wave function $\chi(0) \to 0$ when $k_F \to 0$. In the other weak-coupling limit when $k_F = 1.74$ fm^{-1}, i.e., where the potential changes sign, $v(k_F) = 0$, which inserted in Eq. (13) shows that the wave function goes to $\chi(0) \to 0$ when $k_F \to 1.74$ fm^{-1}.

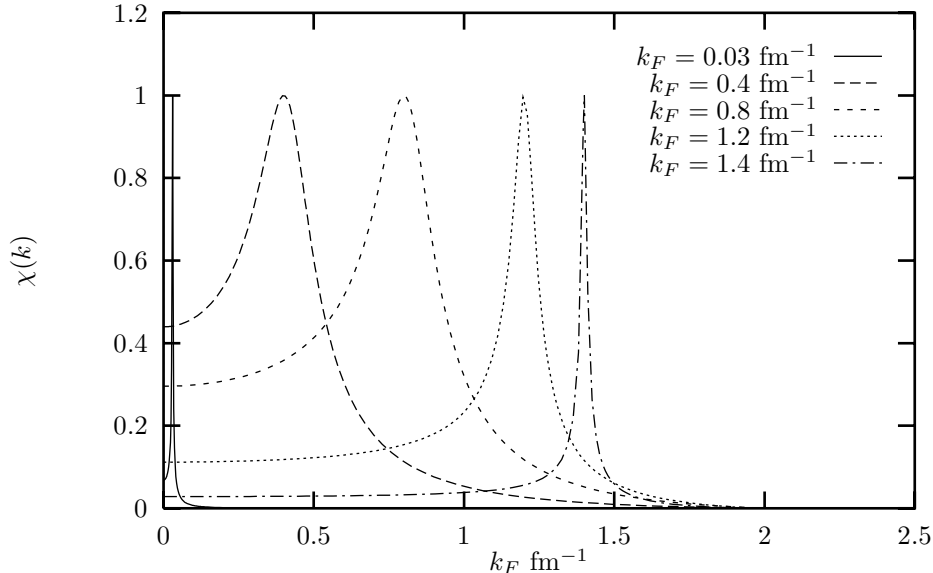
FIG. 3. Wavefunctions in momentum space, $\chi(k)$ for five different values of the Fermi momentum, $k_F = 0.03$ fm^{-1}, $k_F = 0.4$ fm^{-1}, $k_F = 0.8$ fm^{-1}, $k_F = 1.2$ fm^{-1} and $k_F = 1.4$ fm^{-1}.

Finally, at $k = k_F$, one sees from Eq. (12) that $\chi(k = k_F) = 1$, as also seen in Fig. 3.

IV. CONCLUSIONS

In summary, we have shown that in infinite neutron and nuclear matter, owing to the near rank-one separability of the NN interaction in the $^1S_0$ partial wave, we are able to compute the $^1S_0$ pairing gap directly from the NN phase shifts. This explains why all NN potentials which fit the scattering data result in almost identical $^1S_0$ pairing gaps. Our findings conform with the conclusions of Khodel et al. [8] and Carlson et al. [20]: The virtual bound state in $^1S_0$ NN scattering determines the features of nucleon pairing in that partial wave. Even though this result is not likely to survive in a more refined calculation, for instance if one includes polarization effects in the effective pairing interaction like in e.g., Refs. [10,11], one can argue that our results demonstrate that upper limits for the value of the energy gap and for the density where a $^1S_0$ neutron/nucleon superfluid can exist, can be set directly from the $^1S_0$ phase shifts, since the polarization term serves to cut down the value of the gap, and leave the upper density for this superfluid more or less unchanged. These polarization terms will also enhance the already large coherence lengths for the singlet pairs, and one can conclude that for singlet pairing in neutron or nuclear matter, the gas of Cooper pairs will be surrounded by unpaired fermions, the typical coherence length is large compared with the interparticle spacing, and the bound pairs overlap.

Finally, we note that fact that a bound state or a virtual bound state can be used to determine the properties of pairing in a physical system, may be of use in studies of superfluidity and superconductivity in atomic gases, such as a spin-polarized ^{6}Li gas, recently studied by Stoof et al. in [21]. The scattering length of lithium is large and negative, as is the case for the $^1S_0$ state discussed here. Since this is a very dilute system one can then even use an effective range approach to the inter-particle interaction and determine the gap uniquely for such dilute systems, by simply employing a separable interaction of the form shown in Eq. (7) and discussed in Fig. 1.

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